

Six cards are labeled 1, 2, 2, 3, 3, 4, respectively. Considering this deck as the “population,” the population mean is $\mu = 2.5$. Someone who does not know what numbers are written on the cards is going to take a random sample of four cards (without replacement), and will use either the sample mean or the sample midrange to estimate the population mean. (The midrange is defined as $\frac{\text{maximum} + \text{minimum}}{2}$.) We concern ourselves here with the choice of the mean or the midrange for this purpose.

We will start by writing the cards as $\{1, 2, 2^*, 3, 3^*, 4\}$, in order to distinguish the two 2’s and the two 3’s in the population.

- (a) The table below lists all possible samples of size four, along with the value of the sample mean for each sample, and some values of the sample midrange.

| Sample | Sample Mean | Sample Midrange |
|--------------|-------------|-----------------|
| 1, 2, 2*, 3 | 2 | 2 |
| 1, 2, 2*, 3* | 2 | 2 |
| 1, 2, 2*, 4 | 2.25 | 2.5 |
| 1, 2, 3, 3* | 2.25 | |
| 1, 2, 3, 4 | 2.5 | |
| 1, 2, 3*, 4 | 2.5 | |
| 1, 2*, 3, 3* | 2.25 | |
| 1, 2*, 3, 4 | 2.5 | |
| 1, 2*, 3*, 4 | 2.5 | |
| 1, 3, 3*, 4 | 2.75 | |
| 2, 2*, 3, 3* | 2.5 | |
| 2, 2*, 3, 4 | 2.75 | |
| 2, 2*, 3*, 4 | 2.75 | |
| 2, 3, 3*, 4 | 3 | |
| 2*, 3, 3*, 4 | 3 | 3 |

Complete the table by inserting the remaining values of the sample midrange.

(b) The probability distribution for the sample mean is given below.

| | | | | | |
|-------------|----------------|---------------|---------------|---------------|----------------|
| Sample Mean | 2 | 2.25 | 2.5 | 2.75 | 3 |
| Probability | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{1}{5}$ | $\frac{2}{15}$ |

Construct the equivalent probability distribution for the sample midrange.

(c) Calculate the expected value of the sample mean and the expected value of the sample midrange.

- (d) What does the expected value of the sample mean tell you about the appropriateness (or not) of using the sample mean for estimating the population mean?

What does the expected value of the sample midrange tell you about the appropriateness (or not) of using the sample midrange for estimating the population mean?

Explain your answers.

- (e) You are given that the standard deviation of the sample mean is 0.303 and the standard deviation of the sample midrange is 0.365. As an estimator for the population mean, would you choose the statistic with the larger standard deviation or the statistic with the smaller standard deviation? Explain the reasoning behind this choice.