

Show all work.

1. A slot machine has one to nine odds of winning a prize on any given play
  - a) Probability of a win is \_\_\_\_\_.
  - b) Explain why this is a binomial setting if you are told you will play ten times and that  $X$  = the number of wins.
  - c) Assume you play 40 times. Find the mean and standard deviation for this set-up.
  - d) Assume you play 40 times. Find each:
    - i.  $P(X = 5)$
    - ii.  $P(X \leq 8)$
    - iii.  $P(X = 3)$
    - iv.  $P(2 \leq X \leq 9)$
    - v.  $P(2 < X < 9)$
    - vi. The probability that your number of wins is within one standard deviation of the mean.
  - d) Assume  $X$  = number of plays until your first win.  
 $P(X = 6)$   
 $P(X \leq 10)$   
 $P(X = 5)$   
 $P(5 \leq X \leq 10)$   
 $P(X > 5)$

2. A recent study shows the distribution of tickets a driver under twenty has accumulated is a normal distribution with a mean of 3 and a standard deviation of 1.2.
  - a)  $P(X = 3)$
  - b)  $P(X \leq 3.5)$
  - c)  $P(X > 2)$
  - d)  $P(2 \leq X \leq 4)$
  - e)  $P(1.9 < X < 4.3)$

3. A new test for the presence of steroid has a false positive rate of 5%.  
Part I:  $X$  = the number of false positives in a test of eighty students.
  - a)  $P(X = 0)$
  - b)  $P(X \leq 4)$
  - c)  $P(X = 3)$
  - d)  $P(3 \leq X \leq 5)$
  - e)  $P(X \geq 4)$Part II:  $X$  = the number of tests until the first false positive
  - f)  $P(X = 1)$
  - g)  $P(X \leq 5)$
  - h)  $P(X = 15)$
  - i)  $P(1 \leq X \leq 4)$
  - j)  $P(X > 4)$

Part III:

Sketch the probability distribution histogram for the binomial set-up. Label clearly.

4. A shipment is tested for defective parts. A sample of size 125 is taken from a lot. It passes if there are fewer than 3% defective parts. Let  $X$  = the number of defective parts.
- Probability no parts are defective
  - Probability 3% of parts are defective
  - Probability at most 3% of parts are defective
  - If  $Y$  = the number of parts inspected until finding the first defective part,
    - $P(Y = 10)$
    - $P(Y < 20)$
    - $P(Y > 10)$
5. Approximately 36% of the population of Fivonia has type  $O^+$  blood.  $X$  = number of people selected until the first with type  $O^+$  is found. .
- $\mu_x =$
  - $P(X = 3)$
  - $P(X \leq 5)$
  - $P(X \geq 4)$
  - $P(X > 2)$
6. Approximately 47% of the residents of Nordling own a pet. Let  $X$  = the number of pet owners from a sample of 100.
- $\mu_x =$
  - $P(X = 47)$
  - $P(38 \leq X \leq 52)$
  - Probability that at least 47% of the sample are pet owners
7. Explain briefly the difference between the binomial and geometric setting.
8. Approximately 25% of the females in North Pikeville are taller than 70 inches.  
 $X$  = number of females taller than 70 inches in a sample of size 40  
 $Y$  = the number of females selected until the first female taller than 70 inches is selected
- $\mu_x =$   $\sigma_x =$
  - $\mu_y =$
  - $P(Y > 4)$
  - $P(X \geq 25)$

## Binomial

$$P(X=k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$\mu = np$$
$$\sigma = \sqrt{np(1-p)}$$

## Geometric

$$P(X=n) = (1-p)^{n-1} (p)$$

$$\mu = \frac{1}{p}$$
$$\sigma = \sqrt{\frac{1-p}{p^2}}$$

$$P(X > n) = (1-p)^n$$

1. a)  $\frac{1}{10}$  or .1

b) Fixed # events (10)

2 outcomes (success/failure)

independent trials

constant prob. for each trial

c)  $\mu = 40(.1) = 4$

$$\sigma = \sqrt{40(.1)(.9)} = 1.8973$$

d)

i)  $P(X=5) = .16471$

ii)  $P(X \leq 8) = .9845$

iii)  $P(X=3) = .20032$

iv)  $P(2 \leq X \leq 9) = .99494 - .08047 = .91447$

v)  $P(2 < X < 9) = .9845 - .22281 = .76169$

vi)  $P(2.103 \leq X \leq 5.897) = .79373 - .22281 = .57092$

e) Geom.

$$P(X=6) = .05905$$

$$P(X \leq 10) = .6513$$

$$P(X=5) = .06561$$

$$P(5 \leq X \leq 10) = .65132 - .3439 = .3074$$

$$P(X > 5) = .5905 = 1 - \text{geompdf}(.1, 5) = .95$$

See below  
for #2  
Corrected

2.  $\mu = 3$

$\sigma = 1.2$

normal

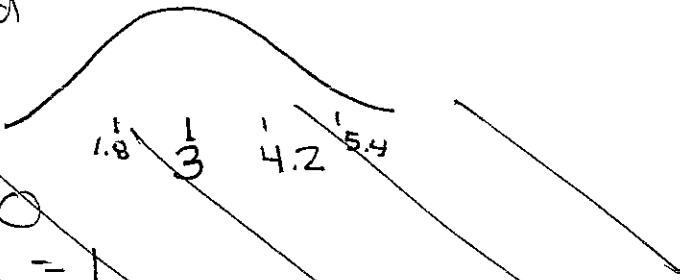
~~$P(X=3) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(3-3)^2}{2(1.2)^2}} = 0$~~

~~$P(X \leq 10) = .999 = 1$~~

~~$P(X=5) = .054$~~

~~$P(5 \leq X \leq 10) = \text{normalcdf}(5, 10, 3, 1.2) = .0478$~~

~~$P(X > 5) = 1 - \text{normalcdf}(5, 1000, 3, 1.2) = .9522$~~



3. Part I - binomial  $n=80, p=.05$

a) .01652

b) .62888

c) .19783

d)  $.78922 - .23062 = .5586$

e)  $1 - .42845 = .57155$

Part II - geom.  $p=.05$

f) .05

g) .22622

h) .02438

i) .18549 - 0

j)  $1 - .18549 = .81451$

Part III -

#2 normalcdf -

a) 0

b) .6615

c) .7977

d) .5953

e) .6810

4) binomial

a) .0222

b)  $P(X = 3\% \text{ of } 125) = P(X = 3.75) = .2$

c)  $P(X \leq 3.75) = .4814$

d) i) .02281

ii) .43939

iii)  $1 - \text{geomcdf}(0.03, 10) = 1 - .26258 = .7374$

5 geom .36

a)  $\mu_x = \frac{1}{.36} = 2.7778$

b) .1475

c) .8926

? d)  $1 - \text{geomcdf}(.36, 3) = .26284$

e)  $P(X > 2) = .64^2 = .4096$

6. Binomial

a)  $\mu_x = .47(100) = 47$

b)  $P(X = 47) = .0797$

c) .8368

d)  $P(X \geq 47) = 1 - .4609 = .5391$

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8. a)  $\mu_x = 40(.25) = 10$

$\sigma_x = \sqrt{40(.25)(.75)} = 2.739$

b)  $\mu_y = .25 = 4$

c)  $P(Y > 4) = .75^4 = .3164$

d)  $P(X \geq 25) = 1 - \text{binomcdf}(40, .25, 24)$   
 $= 5.9 \times 10^{-7} \approx 0$